

## Effects of substrate network topologies on competition dynamics

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We study a competition dynamics, based on the minority game, endowed with various substrate network structures. We observe the effects of the network topologies by investigating the volatility of the system and the structure of follower networks. The topology of substrate structures significantly influences the system efficiency represented by the volatility and such substrate networks are shown to amplify the herding effect and cause inefficiency in most cases. The follower networks emerging from the leadership structure show a power-law incoming degree distribution. This study shows the emergence of scale-free structures of leadership in the minority game and the effects of the interaction among players on the networked version of the game.

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### I. INTRODUCTION

The minority game (MG) captures the features of bounded rationality and inductive reasoning, which are the radically different viewpoint from that of the traditional economics, and has become a representative model of competition dynamics [1–5]. MG has been intensively studied by adopting the agent-based modeling and statistical mechanical approaches analogous to the spin glass theory [6–14]. MG is a repeated game where players choose one out of two alternatives at each time step, which separates the entire group into two subgroups according to the players' choice. The essential rule of MG is that those who happen to be in the minority subgroup with fewer members win each round of the game. Players watch the  $m$  (memory) last winning choices called histories denoted as  $\mu$ , which leads to  $2^m$  possible histories. By taking all histories and assigning choice for each of them, they get a *strategy*. Each player is allowed to have a limited set of  $S$  strategies. She keeps score for each strategy according to the  $m$  last winning choices, i.e., distinguishes “good” and “bad” strategies and chooses the best strategy with the highest score to make a decision for the next step [2].

Even this simplest original model of MG shows quite interesting properties such that the volatility  $\sigma$ , the standard deviation of the time series  $A(t)$  which counts the number of players choosing a certain alternative, follows the scaling relation  $\sigma^2/N=f(2^m/N)$  [6]. Furthermore, this scaling function is divided into two phases called symmetric and asymmetric phases, where the strategy distribution and the system predictability are different [6]. The volatility is considered as an important quantity because it is the measure of the system inefficiency, provided that the maximum number of winners is bounded by  $N/2$ . This behavior of the volatility has, therefore, caught much attention from researchers.

Besides such efforts to unveil the structure and properties of the original MG, some extensions or variations of the original model are introduced to adopt more realistic aspects or for mathematical convenience [7,12]. The concept of so-

cial interaction among players in terms of “imitation,” superimposed with the global information, is introduced in Refs. [15,16]. Later, in Ref. [17], players in MG are connected by the Erdős-Rényi (ER) random graph [18]. In their model, an undirected *substrate* network  $\mathbf{G}$  of the ER type is constructed between players before the game. For each moment of choice, each player first chooses one of two alternatives according to her own best strategy, but she does not use the choice immediately. Instead, she compares the points, defined as the number of victories of each player from the beginning of the game, of her neighbors connected by the substrate network and follows the choice of the player with the maximum points. If no neighbor is better than herself, she makes the final decision by following her own best strategy. After some time, the *leadership structure*, i.e., who follows whom, appears and is a directed subset  $\mathbf{F}$  of the underlying substrate network  $\mathbf{G}$ . Anghel *et al.* shows that despite the Poisson degree distribution of the substrate ER graph, the incoming degrees of this “follower” network  $\mathbf{F}$  are described by a power-law distribution with a sharp cutoff [17], which indicates the *scale-free* structure known to be ubiquitous in the real world network systems [19–21].

In this paper, we generalize the approach of Anghel *et al.* by varying the types of substrate networks, such as the ER random graph [18], regular lattices, small-world networks [22], and scale-free networks [23,24], since the real social network structures are known to be much more complicated than the ER random graph. Furthermore, we observe not only the structure of the follower networks, but also the volatility  $\sigma$ . There have been quite a number of studies that set up “recently recognized” complex network structures in several systems of interacting components, e.g., the Ising model [25–30], the voter model [31–35], the prisoners' dilemma game [36–40], the snowdrift game [40–42], the Boolean game [43], etc. All these works have provided opportunities to understand both the systems themselves and the properties of complex networks better. Similarly, we expect that this work also reveals some features of both sides.

### II. RULES OF THE GAME AND SUBSTRATE NETWORK STRUCTURES

The rules are basically the same as in Ref. [17] as stated in the previous section, except for using various kinds of

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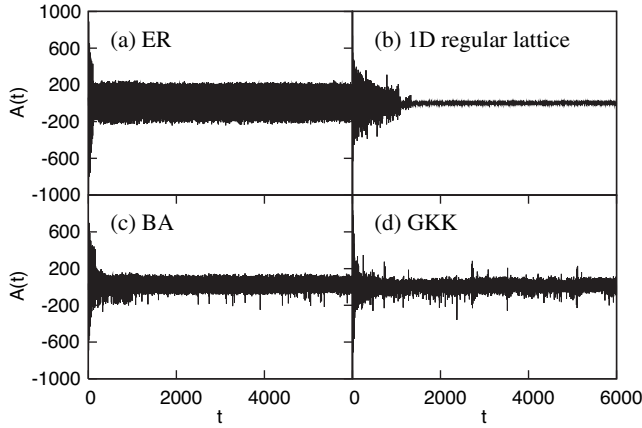


FIG. 1. Typical time series  $A(t)$  with  $N=1001$  and  $m=2$  for (a) ER, (b) 1D regular lattice, (c) BA, and (d) GKK substrate networks.

networks as underlying substrate structures. Each player follows her neighbor with the best performance by imitating the neighbor's action (choice) at each time step, as well as keeping records of her own strategies. We set the number of each player's strategies as  $S=2$  and change the memory  $m$  to observe the behavior of MG systems. Similar to other studies on MG, we confirmed that most qualitative features do not change for other values of  $S$ .

As undirected substrate networks, we use the ER random graph [18], the one-dimensional (1D) regular lattice, where each player is connected to her neighbors two or fewer lattice spacings away, the Watts-Strogatz (WS) small-world net-

work [22], the Barabási-Albert (BA) growing scale-free network with the degree exponent  $\gamma=3$  where the degree distribution  $p(k) \sim k^{-\gamma}$  [23], and the Goh-Kahng-Kim (GKK) static scale-free network with  $\gamma=2$  [24]. For the volatility function, especially we focus on the WS case, varying the rewiring probability  $p$ , since the volatility function changes its shape continuously but drastically with the disorder parameter  $p$ . In addition, we try some real-world social networks such as a coauthorship network [44] and bulletin board system (BBS) networks, composed of BBS users connected by their replies among each other [45], to test the generality of the results.

### III. VOLATILITY FUNCTION

The term volatility  $\sigma$  in MG is defined as the standard deviation of the time series  $A(t)$ , representing the difference between the number of agents who choose a certain alternative and the other at time  $t$ , as follows:

$$\sigma = \sqrt{\langle A^2(t) \rangle - \langle A(t) \rangle^2}. \quad (1)$$

Here the angular bracket refers to the average over time. If  $A(t)$  is a random sequence of two alternatives with equal probabilities,  $\sigma^2/N=1/4$ , and the situation is the so-called "coin-toss" case. Minimizing this volatility corresponds to maximizing the efficiency of the system, since the amount of deviation from the average value  $N/2$  (by symmetry) is viewed as the *waste* of available resources. In this sense, the volatility is a reciprocal measure of the system efficiency.

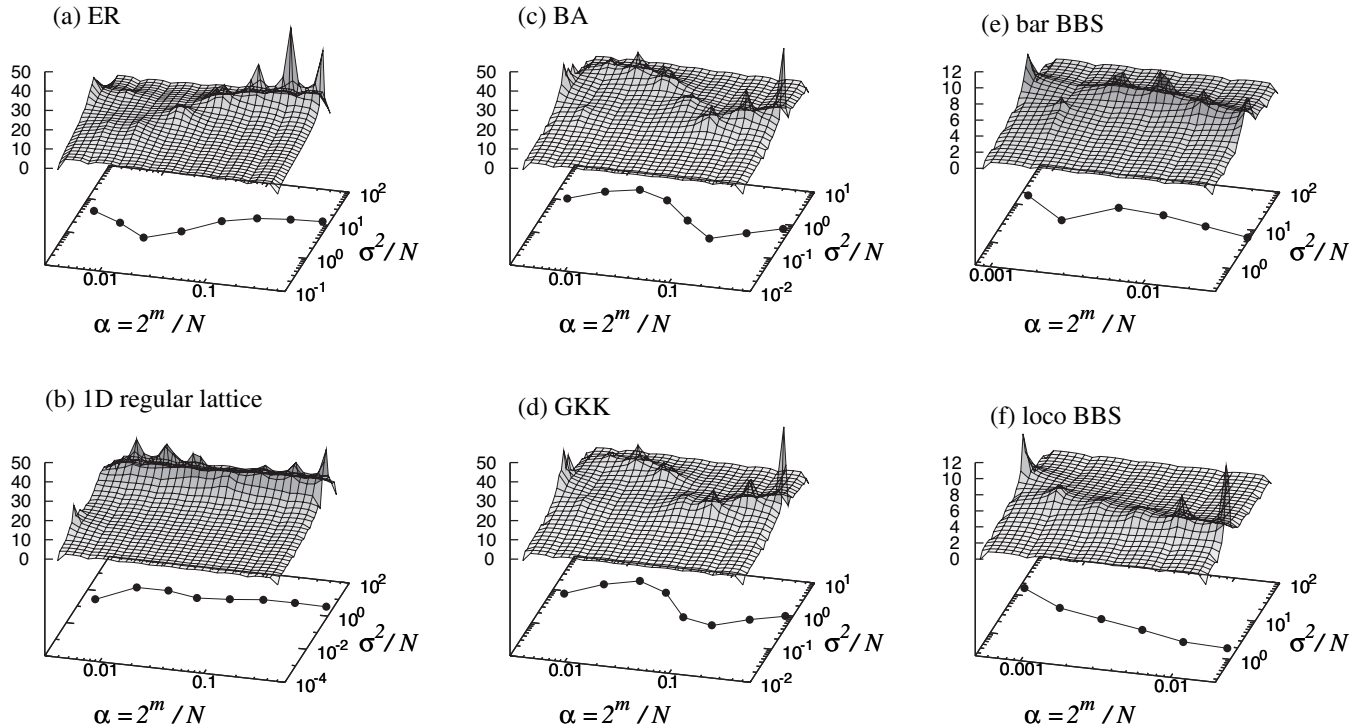


FIG. 2. Volatility  $\sigma^2/N$  distribution according to the value  $\alpha=2^m/N$ , for (a) ER, (b) 1D regular lattice, (c) BA, (d) GKK, (e) bar BBS, and (f) loco BBS substrate networks. The numbers of players are  $N=1001$  for (a)–(d),  $N=4461$  for (e), and  $N=7410$  for (f). For each value of memory  $m$ , the results of 500 simulations for (a)–(d) are recorded as a histogram, whereas (e) and (f) are from 100 realizations. Average values of the volatility are drawn at the bottom of each graph.

A remarkable result of early studies on the original MG model is that  $\sigma^2/N$  does not depend independently on  $m$  and  $N$ , but only on the ratio  $\alpha=2^m/N$  [6]. This scaling relation is explained as the effect caused by the relative size difference in the number of player  $N$  and the size of *effective* strategy space  $2^m$ , related to the distribution and overlap of strategies among players. For  $\alpha \ll 1$ ,  $\sigma^2 \sim 1/\alpha$  and this phase is called a crowded, unpredictable, and symmetric phase. On the other hand, when  $\alpha \gg 1$ ,  $\sigma^2 \sim 1/4$  (the system approaches the coin-toss case) and this phase is called an uncrowded, predictable, and asymmetric phase. The phase separation has been investigated by analogy with spin formalism [11,13,14].

Extensive simulations of MG on various substrate networks show that in contrast to the case of the original MG, where the clear scaling behavior of volatility is shown, the volatility  $\sigma^2/N$  is not a function of the single variable  $\alpha=2^m/N$ . Moreover, variance of  $\sigma^2/N$ , for each realization, is significantly large for some cases. Figure 1 shows typical times series for various substrate networks and the volatility is shown in Fig. 2. For the ER substrate network, the form of volatility is similar to the original MG model. Other topologies, however, totally change the shape of the volatility function against  $\alpha$ . For a 1D regular lattice, except for the very small value of  $\alpha$ , the volatility is so large that the system can be considered as inefficient or maladapted. In the case of a very small value of  $\alpha$ , on the other hand, the volatility becomes quite small [46]. [Notice the scale of the volatility axis in Fig. 2(b).] For  $N=1001$ , extremely small volatility shown in Fig. 1(b) happens only for  $m=2$ , the smallest value of  $m$  used in the simulation, and increasing the value just to  $m=3$  causes inefficiency. The volatility for  $m=2$  in this case (1D regular lattice) is observed as the smallest volatility among all the simulation results. Therefore, from the perspective of the system efficiency defined as the reciprocal of the volatility, a 1D regular lattice as substrate and a very small value of  $\alpha$  are the best ingredients for an efficient system.

For BA and GKK substrate networks, the shapes of volatility against  $\alpha$  quite resemble each other, in spite of the difference in the creation mechanism [23,24] and the degree exponent  $\gamma$ . [Compare Figs. 2(c) and 2(d).] The peculiar part, compared with the original MG or MG on ER substrate network, is the increasing behavior of volatility for small  $\alpha$ . Since MG stems from economics, where social networks among people are important, we also use BBS networks in a university [45] as examples of real social networks for the simulation. The two BBS networks have their own characteristic shapes of the volatility as shown in Figs. 2(e) and 2(f). From the simulation result, it is clear that the topology of substrate networks significantly influences the dependence of system volatility on the parameter  $\alpha$ . Especially, there is a drastic difference between the ER random graph and 1D regular lattice because the randomness or disorderedness is extremely different, although both of these two networks have homogeneous degree distributions. The scale-free networks with heterogeneous degree distributions seem to be located somewhere in between those two extremes. The fact that the underlying network topology effects on the ability to communicate and the information transfer has been shown recently [47] and the volatility function of MG on networks here is another good example.

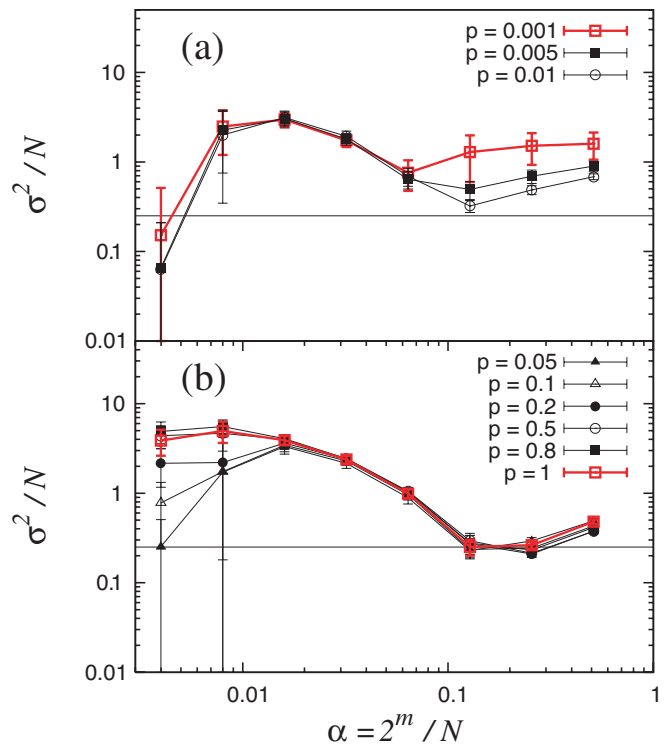


FIG. 3. (Color online) Volatility  $\sigma^2/N$  according to the value  $\alpha=2^m/N$  for WS small-world network with (a) small, and (b) large values of rewiring probability  $p$ . The number of players is  $N=1001$ , and each player is connected to her neighbors two or fewer lattice spacings away. For each value of memory  $m$ , the results of 500 simulations are averaged. The solid lines are guides to the eyes, and the horizontal line ( $\sigma^2/N=0.25$ ) indicates the coin-toss case. Each (red) line corresponds to a limiting case of  $p$ , i.e., the 1D regular lattice in (a) and the ER random graph in (b), respectively.

For the more systematic approach, we use the WS small-world network [22]. One convenient aspect about the WS network is that we can control the disorderedness of the structure by changing the rewiring probability  $p$ . Figure 3 shows the result of volatility from MG on WS networks with different values of  $p$ . By definition of the rewiring probability  $p$ , the case of  $p \rightarrow 0$  ( $p \rightarrow 1$ ) corresponds to the 1D regular lattice (ER random network), respectively. As one can imagine, increasing  $p$  from 0 to 1 (adding disorderedness to the substrate network) continuously changes the form of volatility from the 1D regular lattice case to the ER random network case. Figures 3(a) and 3(b), furthermore, show a certain pattern of change depending on the value of  $p$ . For small values of  $p$  [Fig. 3(a)], the volatility for large values of  $\alpha$  changes rapidly as  $p$  increases. On the other hand, for large values of  $p$  [Fig. 3(b)], the volatility for small values of  $\alpha$  changes rapidly as  $p$  increases, while that for large  $\alpha$  is almost fixed.

The region of small values of  $\alpha=2^m/N$  is traditionally called the symmetric or crowded phase in the original MG, which means that the number of players  $N$  is much larger than the effective size of the strategy space, so that many players come to use the same strategies. Large values of  $\alpha$  correspond to the asymmetric, anticrowded, or *information-*

*rich* phase, because the lack of overusage of strategies causes inequality of strategies' scores in the whole system, which leads to the possibility of exploiting the information and predicting. According to the properties of WS small-world networks, the average path length of the network drops dramatically as  $p$  increases, even for small values of  $p$ , while the clustering coefficient finally drops for large values of  $p$ . Therefore, we can conclude that for small values of  $p$ , a radically decreasing average path length, possibly related to the efficiency of the information transfer, affects the information-rich phase, i.e., the anticrowded phase for the large value of  $\alpha$ . For large values of  $p$ , a sharply dropping value of clustering coefficient seems to affect the crowded effect for the crowded phase with a small value of  $\alpha$ . From this result, we can see that the disorderedness of the substrate networks clearly influences the system behavior, in regard to the volatility.

It is worthwhile to note that for most regions of parameters and types of substrate networks, the volatility is *larger* than that in the original MG and even the random coin-toss case. In other words, the system with networks performs *less* effectively in general. The reason why the original MG system cannot reach a Nash equilibrium, i.e.,  $\sigma \sim \mathcal{O}(1)$ , is that each player does not consider her own "market impact" on the game and causes the herding behavior [48]. Adding substrate networks, which enable players to interact with one another, even more accelerates this herding effect thanks to the locally common references and leads to the inefficiency. Basically, each player plays the game without any *centralized* control and this sort of inefficiency called "price of anarchy," due to such decentralization, has gotten attention recently [49]. An onset of bubble phenomena by rumors from peers is not unusual in economy, which is a good example of the social network effect.

Figure 4(a) shows that a "hub" with many connections in the first place is likely to gain a lower score than other players, due to the herding behavior of the players following the hub. A hub tends to be more likely to change its outgoing link in the follower network during a game, as shown in Fig. 4(b). We can easily see that a player's score and the frequency of the outgoing link change are closely related as well, from Fig. 4(c). In sum, hubs are likely to lose due to their followers, which leads to the frequent changes of their outgoing links in the follower network. For many other networks and parameters, this pattern is observed in common. Players with large degrees, therefore, are not able to take advantage of the situation. Oppositely, they suffer from the lowered chance to win because of their followers who will make the same choice with them, which of course the hub players do not want, considering the very definition of the minority rule.

Besides the volatility, the predictability  $H$  is also an important quantity in the MG system and characterizes the phase transition. The predictability is defined as [4,5],

$$H = \frac{1}{2^m} \sum_{\mu} \langle A | \mu \rangle^2, \quad (2)$$

where  $\mu$  is the  $m$ -bit history, the summation over  $\mu$  is for all the possible  $2^m$  values of  $\mu$ , and  $\langle A | \mu \rangle$  is the temporal aver-

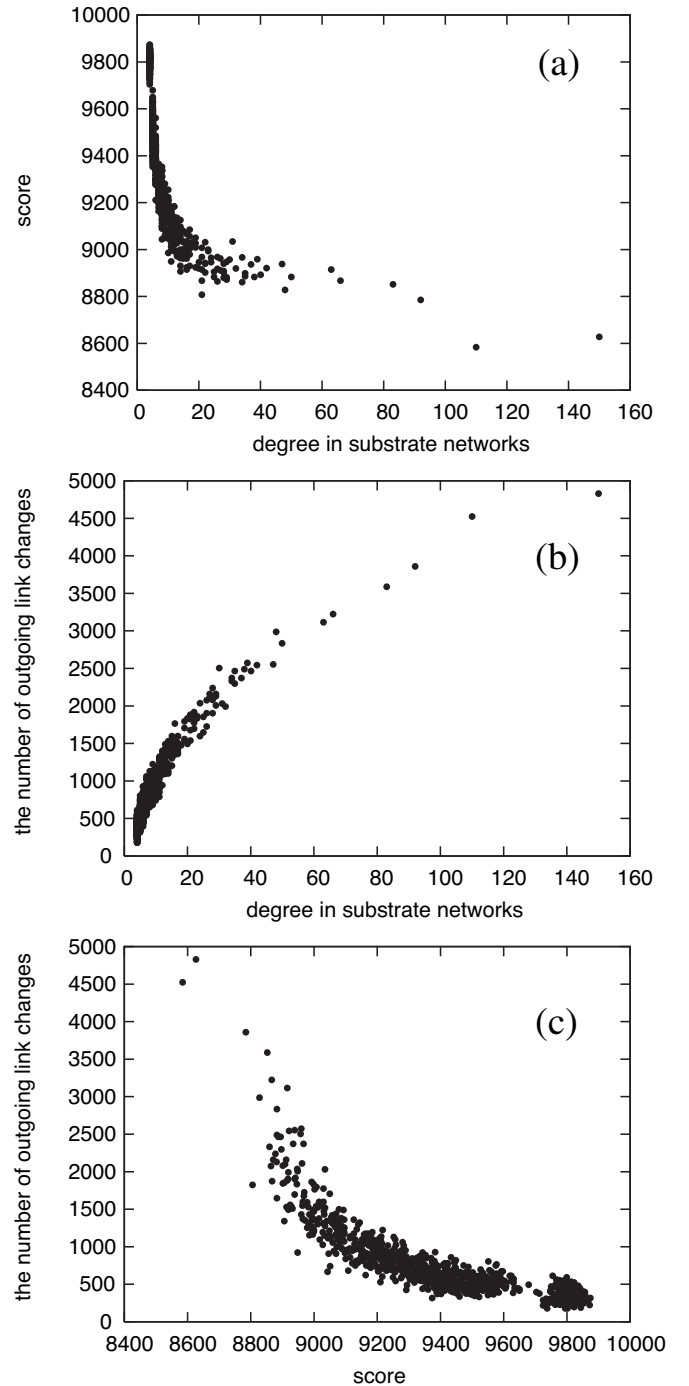


FIG. 4. The interrelationship among the average scores of players, their degrees in the substrate network, and the average numbers of outgoing link changes per game. Each data point represents the values corresponding to each player averaged over 500 games, each of which is composed of 20 000 steps. ( $m=2$ ) The substrate network is the BA model with 1001 nodes.

age value of the outcomes of the game for each history  $\mu$ , composed of the last  $m$  bits of winning choice. The predictability, therefore, measures the "deterministic" tendency of the game for each given history. The larger values of the predictability implies that the outcome of the game at each time is more predictable and each player tends to stick to a

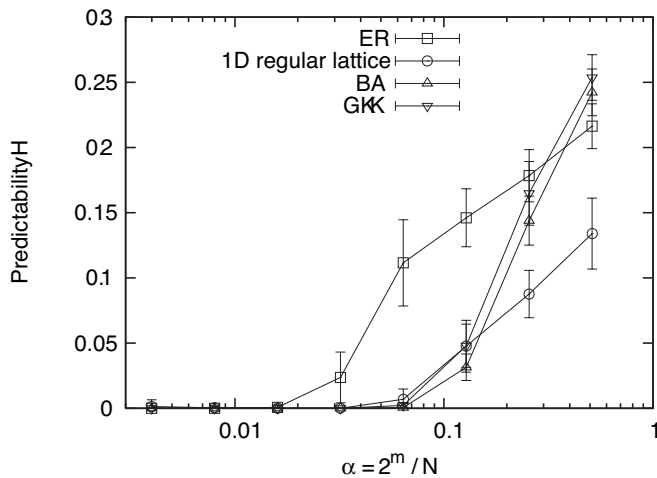


FIG. 5. Predictability  $H$  according to the value  $\alpha=2^m/N$ , for each substrate network whose size  $N=1001$ . Each point and error bar are drawn values averaged over 100 games, each of which is composed of 20 000 steps.

single strategy, which gives the analogy of the ordered spin state in the spin-glass theory.

In the original MG model, the predictability is in the vicinity of 0 at the symmetric phase (corresponding to small  $\alpha=2^m/N$  values) and starts to increase with the onset of the asymmetric phase as  $\alpha$  increases. This characteristic of  $H$  according to  $\alpha$  is the reason why the symmetric phase is called the unpredictable phase and the asymmetric phase is called the predictable phase. Figure 5 shows the predictability values according to  $\alpha$  for our MG model with various substrate network structures. Like the original MG model, the predictability starts to increase from a certain threshold value of  $\alpha$ , below which the predictability is essentially 0. The threshold values, however, depend on substrate network topologies. Comparing Figs. 2(a)–2(d) and 5, we can see that, for MG on ER random networks, the volatility reaches its minimum value and the predictability begins to increase for smaller value of  $\alpha$  than for the game on the other networks.

#### IV. STRUCTURES OF THE FOLLOWER NETWORKS

After the transient period, the follower network structure reaches its steady state and only a small fraction of nodes change their outgoing link at each time step (less than ten percent even for the worst case). In addition, even though some nodes change their outgoing link, the characteristic features such as the degree distribution do not change. Therefore, we analyze the structure of the follower networks for each substrate network topology.

Figure 6 shows a typical structure of follower networks for the case of the ER substrate network [50]. If node  $a$  follows node  $b$ , the direction of link  $a \rightarrow b$  is established. Hubs with large incoming degree (in degree) are easily noticed. In-degree distribution of follower networks shown in Fig. 7 implies the scale-free structure of follower networks for (a) ER, (b) BA, and (c) GKK substrate networks. We also

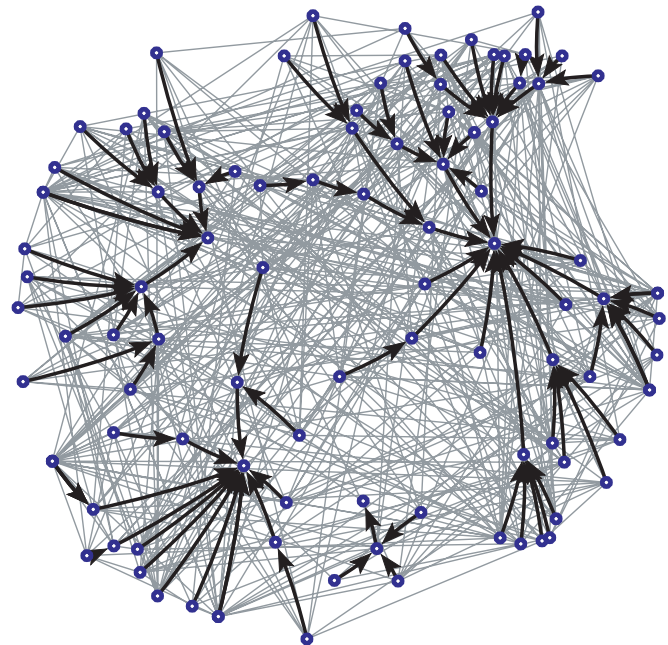


FIG. 6. (Color online) Snapshot of typical structure of follower network, where the ER random graph is used as substrate in this case. Thin gray lines stand for the substrate network and thick black arrows for the follower network.

observed the same results of the fat-tailed in-degree distributions for the e-print archive coauthorship network [44] and BBS networks [45], as shown in Fig. 8. Cutoffs in Fig. 7(a), also stated in Ref. [17], are due to the fact that the number of links for each node is bounded by the Poisson degree distribution of the substrate network. In this respect, the scale-free structure is conserved for MG. In addition, from Figs. 7 and 8, we can see that the larger the memory size  $m$  is, the narrower the distribution gets. In other words, larger memory sizes of individual players tend to lower the possibility of the emergence of large hubs with many connections. Interestingly, this phenomenon can be interpreted in that “smarter” players effectively prevent the appearance of “dictators” with big power.

#### V. DISCUSSION AND CONCLUSIONS

Even though both MG and complex networks have been heavily studied to understand the structure and dynamics of complex systems, the interaction among players in the MG system via complex networks has not drawn much attention so far. By numerically studying this subject, we have shown that the topology of underlying substrate networks significantly influences the well-known properties of MG. We also demonstrate the emergence of scale-free coordination structures, as suggested in earlier studies.

Volatility, traditionally considered as the reciprocal measure of the system efficiency, of MG on various networks has quite different structures from the original MG, depending on the topology of the substrate. Especially, MG on 1D regular lattice with very small memory size seems to exploit the system effectively as shown in Fig. 1(b), although the infor-

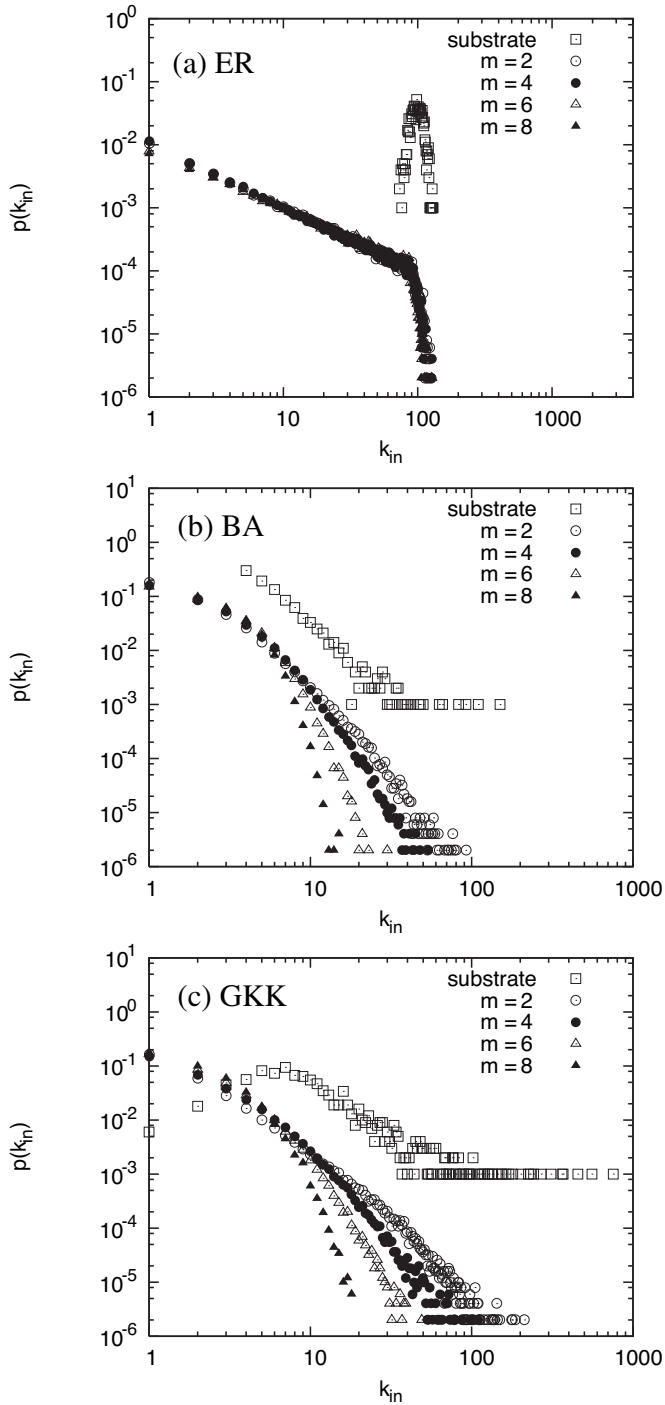


FIG. 7. Incoming degree distribution of the follower networks for  $N=1001$  whose substrate networks are (a) the ER random graph with connecting probability  $p=0.1$  [18], (b) the BA model with  $m_0=m=4$  [23], and (c) the GKK model with  $\gamma=2$  and average number of degrees=10 [24]. Empty squares ( $\square$ ) correspond to the degrees of substrate networks. The degree distribution of each follower network is drawn from 500 realizations of MG.

mation transfer is known not to be efficient in regular lattices. In contrast, in most cases the performance of systems is worse than the original MG or the random choice, due to the enhanced herding behavior provided by interactions via substrate networks. Disorderedness of the substrate

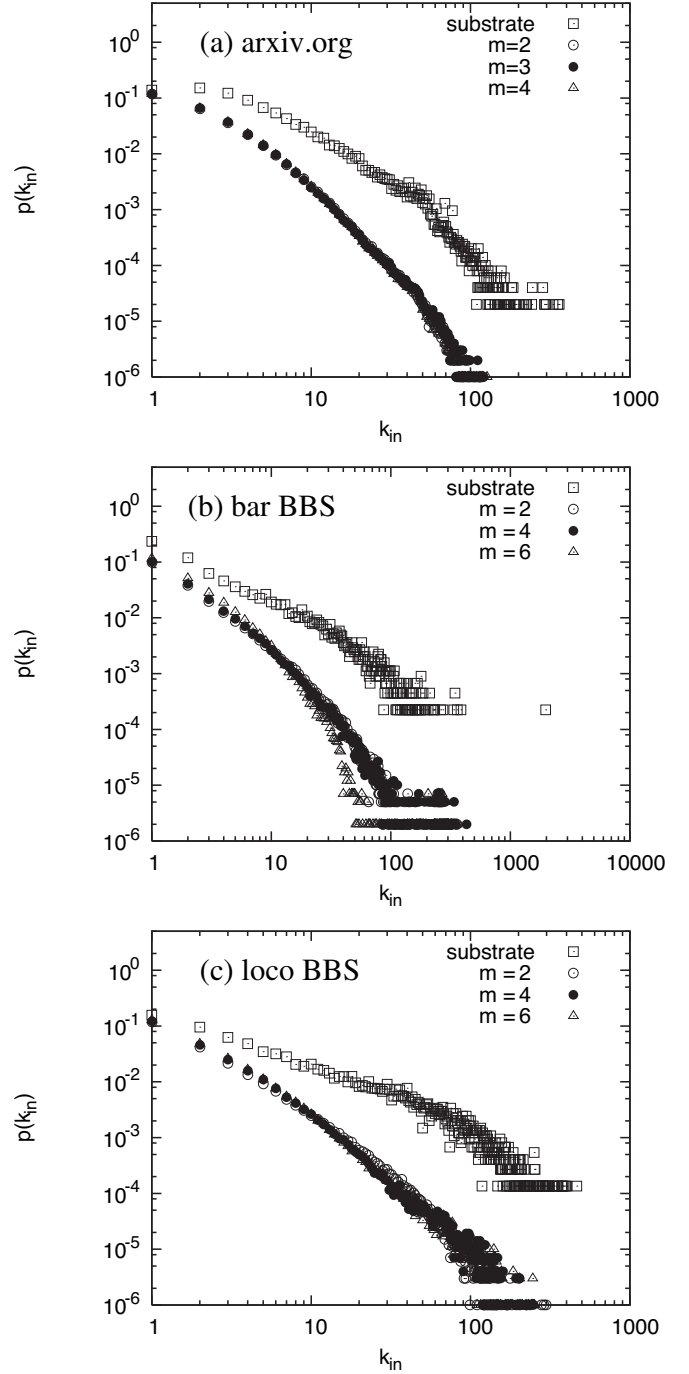


FIG. 8. Incoming degree distribution of the follower networks for whose substrate networks are (a) the e-print archive coauthorship network with 49 983 nodes [44], (b) the bar BBS network with 4461 nodes [45], and (c) the loco BBS network with 7410 nodes [45]. Empty squares ( $\square$ ) correspond to the degrees of substrate networks. The degree distribution of each follower network is drawn from 100 realizations of MG.

networks, reflected by the rewiring probability  $p$  of the WS small-world network, affects the form of the volatility function in different domains, depending on the degree of disorderedness.

MG is famous for its simple rules, and at the same time for its complex structure including many counterintuitive as-

pects. We have shown that the properties of substrate interaction topology, such as the degree distribution and randomness, play a great role in the system. Still, there are many open questions about the qualitative and quantitative properties of this system. The original MG has been investigated not only by numerical simulation, but also by analytic approaches which have accelerated the understanding of this complex game. The analytic approach of MG with interactions among players has also started [17], and we hope that more research on this subject of the emergence of complex

behavior, including MG with interactions, will be continued in the future.

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